10.5 Apply Other Angle Relationships in Circles

Before You found the measures of angles formed on a circle.

Now You will find the measures of angles inside or outside a circle.

Why So you can determine the part of Earth seen from a hot air balloon, as in Ex. 25.

Key Vocabulary
- **chord,** *p. 651*
- **secant,** *p. 651*
- **tangent,** *p. 651*

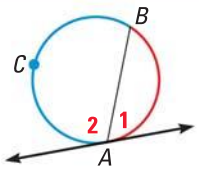
You know that the measure of an inscribed angle is half the measure of its intercepted arc. This is true even if one side of the angle is tangent to the circle.

THEOREM *For Your Notebook*

THEOREM 10.11

If a tangent and a chord intersect at a point on a circle, then the measure of each angle formed is one half the measure of its intercepted arc.

Proof: Ex. 27, p. 685

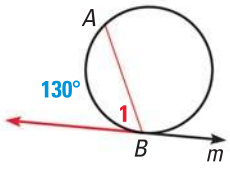


$m\angle 1 = \frac{1}{2}m\overset{\frown}{AB}$ $m\angle 2 = \frac{1}{2}m\overset{\frown}{BCA}$

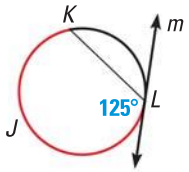
EXAMPLE 1 Find angle and arc measures

Line *m* is tangent to the circle. Find the measure of the red angle or arc.

a.



b.



Solution

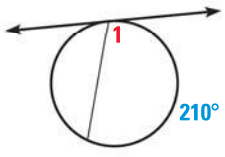
a. $m\angle 1 = \frac{1}{2}(130°) = 65°$

b. $m\overset{\frown}{KJL} = 2(125°) = 250°$

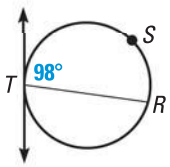
GUIDED PRACTICE for Example 1

Find the indicated measure.

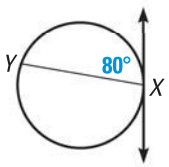
1. $m\angle 1$



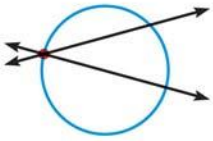
2. $m\overset{\frown}{RST}$



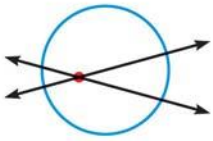
3. $m\overset{\frown}{XY}$



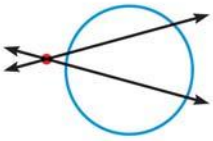
INTERSECTING LINES AND CIRCLES If two lines intersect a circle, there are three places where the lines can intersect.



on the circle



inside the circle



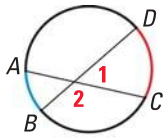
outside the circle

You can use Theorems 10.12 and 10.13 to find measures when the lines intersect *inside* or *outside* the circle.

THEOREMS *For Your Notebook*

THEOREM 10.12 Angles Inside the Circle Theorem

If two chords intersect *inside* a circle, then the measure of each angle is one half the *sum* of the measures of the arcs intercepted by the angle and its vertical angle.



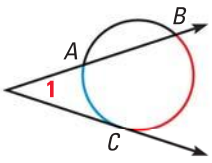
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{DC} + m\overset{\frown}{AB})$,

$m\angle 2 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

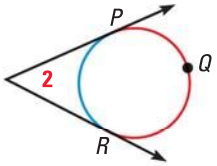
Proof: Ex. 28, p. 685

THEOREM 10.13 Angles Outside the Circle Theorem

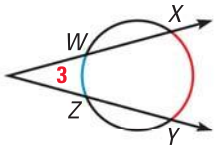
If a tangent and a secant, two tangents, or two secants intersect *outside* a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.



$m\angle 1 = \frac{1}{2}(m\overset{\frown}{BC} - m\overset{\frown}{AC})$



$m\angle 2 = \frac{1}{2}(m\overset{\frown}{PQR} - m\overset{\frown}{PR})$

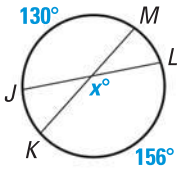


$m\angle 3 = \frac{1}{2}(m\overset{\frown}{XY} - m\overset{\frown}{WZ})$

Proof: Ex. 29, p. 685

EXAMPLE 2 Find an angle measure inside a circle

Find the value of x.



Solution

The chords $\overline{JL}$ and $\overline{KM}$ intersect inside the circle.

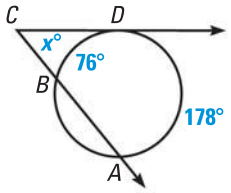
$x^\circ = \frac{1}{2}(m\overset{\frown}{JM} + m\overset{\frown}{LK})$ **Use Theorem 10.12.**

$x^\circ = \frac{1}{2}(130^\circ + 156^\circ)$ **Substitute.**

$x = 143$ **Simplify.**

EXAMPLE 3 Find an angle measure outside a circle

Find the value of x.



Solution

The tangent $\overrightarrow{CD}$ and the secant $\overrightarrow{CB}$ intersect outside the circle.

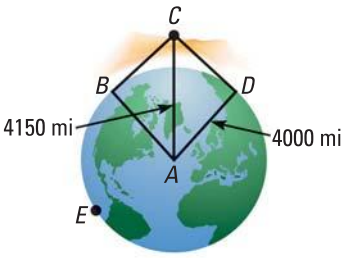
$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{AD} - m\overset{\frown}{BD}\right) \quad \text{Use Theorem 10.13.}$$

$$x^\circ = \frac{1}{2}(178^\circ - 76^\circ) \quad \text{Substitute.}$$

$$x = 51 \quad \text{Simplify.}$$

EXAMPLE 4 Solve a real-world problem

SCIENCE The Northern Lights are bright flashes of colored light between 50 and 200 miles above Earth. Suppose a flash occurs 150 miles above Earth. What is the measure of arc BD, the portion of Earth from which the flash is visible? (Earth's radius is approximately 4000 miles.)



Not drawn to scale

Solution

Because $\overline{CB}$ and $\overline{CD}$ are tangents, $\overline{CB} \perp \overline{AB}$ and $\overline{CD} \perp \overline{AD}$. Also, $\overline{BC} \cong \overline{DC}$ and $\overline{CA} \cong \overline{CA}$. So, $\triangle ABC \cong \triangle ADC$ by the Hypotenuse-Leg Congruence Theorem, and $\angle BCA \cong \angle DCA$. Solve right $\triangle CBA$ to find that $m\angle BCA \approx 74.5^\circ$. So, $m\angle BCD \approx 2(74.5^\circ) \approx 149^\circ$. Let $m\overset{\frown}{BD} = x^\circ$.

$$m\angle BCD = \frac{1}{2}\left(m\overset{\frown}{DEB} - m\overset{\frown}{BD}\right) \quad \text{Use Theorem 10.13.}$$

$$149^\circ \approx \frac{1}{2}[(360^\circ - x^\circ) - x^\circ] \quad \text{Substitute.}$$

$$x \approx 31 \quad \text{Solve for } x.$$

▶ The measure of the arc from which the flash is visible is about 31°.

AVOID ERRORS Because the value for $m\angle BCD$ is an approximation, use the symbol $\approx$ instead of $=$.

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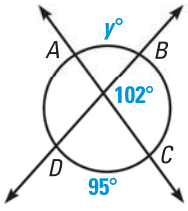
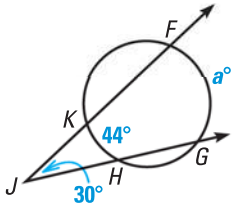
GUIDED PRACTICE for Examples 2, 3, and 4

Find the value of the variable.

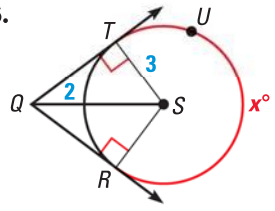
4.



5.



6.



10.5 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 3, 9, and 23

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 13, 15, 19, and 26

SKILL PRACTICE

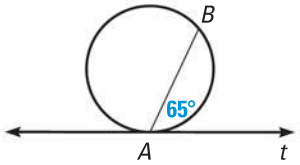
1. **VOCABULARY** Copy and complete: The points A, B, C, and D are on a circle and $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at P. If $m\angle APC = \frac{1}{2}(m\overset{\frown}{BD} - m\overset{\frown}{AC})$, then P is __?__ (*inside*, *on*, or *outside*) the circle.

2. ★ **WRITING** What does it mean in Theorem 10.12 if $m\overset{\frown}{AB} = 0°$? Is this consistent with what you learned in Lesson 10.4? *Explain* your answer.

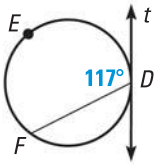
EXAMPLE 1 on p. 680 for Exs. 3–6

FINDING MEASURES **Line t is tangent to the circle. Find the indicated measure.**

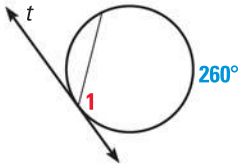
3. $m\overset{\frown}{AB}$



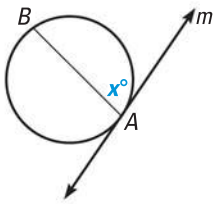
4. $m\overset{\frown}{DEF}$



5. $m\angle 1$



6. ★ **MULTIPLE CHOICE** The diagram at the right is not drawn to scale. $\overline{AB}$ is any chord that is not a diameter of the circle. Line m is tangent to the circle at point A. Which statement must be true?



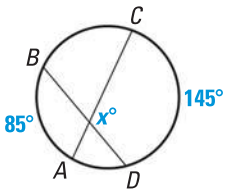
(A) $x \leq 90$ (B) $x \geq 90$

(C) $x = 90$ (D) $x \neq 90$

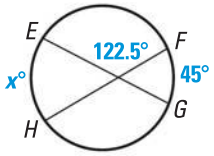
FINDING MEASURES **Find the value of x.**

EXAMPLE 2 on p. 681 for Exs. 7–9

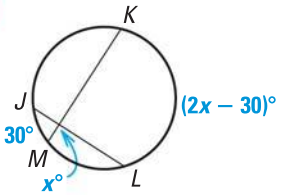
7.



8.

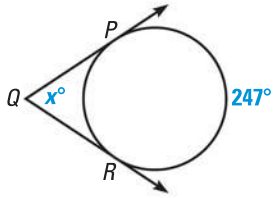


9.

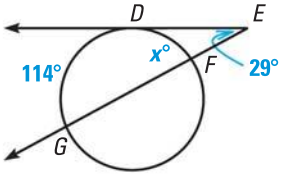


EXAMPLE 3 on p. 682 for Exs. 10–13

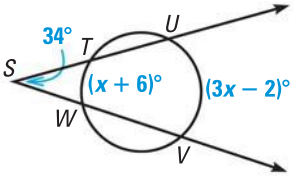
10.



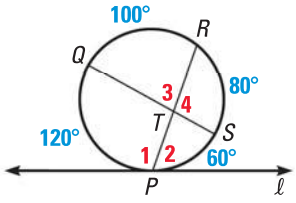
11.



12.



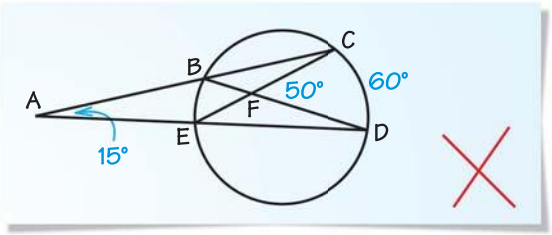
13. ★ **MULTIPLE CHOICE** In the diagram, ℓ is tangent to the circle at P. Which relationship is not true?



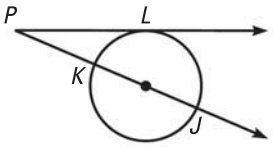
(A) $m\angle 1 = 110°$ (B) $m\angle 2 = 70°$

(C) $m\angle 3 = 80°$ (D) $m\angle 4 = 90°$

14. ERROR ANALYSIS *Describe* the error in the diagram below.

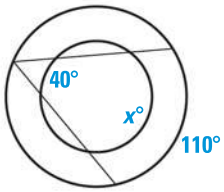


15. ★ SHORT RESPONSE In the diagram at the right, $\overrightarrow{PL}$ is tangent to the circle and $\overline{KJ}$ is a diameter. What is the range of possible angle measures of $\angle LPJ$? *Explain*.

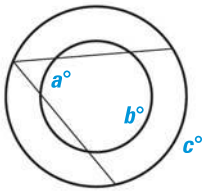


16. CONCENTRIC CIRCLES The circles below are concentric.

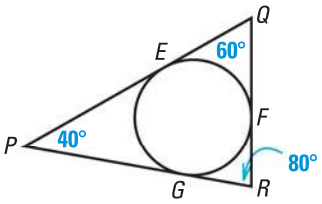
a. Find the value of x.



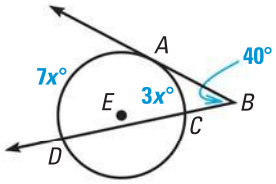
b. Express c in terms of a and b.



17. INSCRIBED CIRCLE In the diagram, the circle is inscribed in $\triangle PQR$. Find $m\widehat{EF}$, $m\widehat{FG}$, and $m\widehat{GE}$.



18. ALGEBRA In the diagram, $\overrightarrow{BA}$ is tangent to $\odot E$. Find $m\widehat{CD}$.



19. ★ WRITING Points A and B are on a circle and t is a tangent line containing A and another point C.

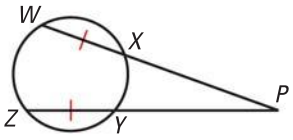
a. Draw two different diagrams that illustrate this situation.

b. Write an equation for $m\widehat{AB}$ in terms of $m\angle BAC$ for each diagram.

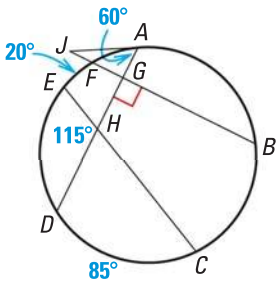
c. When will these equations give the same value for $m\widehat{AB}$?

CHALLENGE Find the indicated measure(s).

20. Find $m\angle P$ if $m\widehat{WZY} = 200°$.



21. Find $m\widehat{AB}$ and $m\widehat{ED}$.

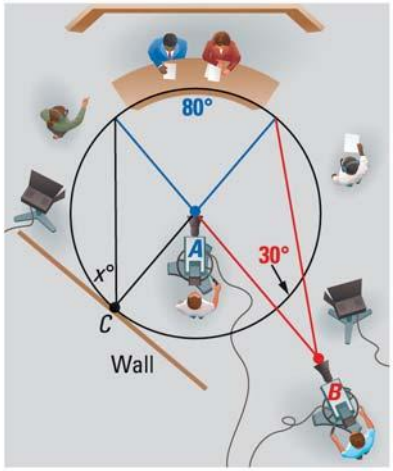


○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

VIDEO RECORDING **In the diagram at the right, television cameras are positioned at *A*, *B*, and *C* to record what happens on stage. The stage is an arc of ⊙*A*. Use the diagram for Exercises 22–24.**



22. Find $m\angle A$, $m\angle B$, and $m\angle C$.

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23. The wall is tangent to the circle. Find x without using the measure of $\angle C$.

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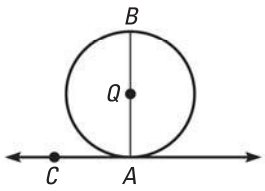
24. You would like Camera *B* to have a 30° view of the stage. Should you move the camera closer or further away from the stage? *Explain*.

EXAMPLE 4 on p. 682 for Ex. 25

25. HOT AIR BALLOON You are flying in a hot air balloon about 1.2 miles above the ground. Use the method from Example 4 to find the measure of the arc that represents the part of Earth that you can see. The radius of Earth is about 4000 miles.

26. ★ EXTENDED RESPONSE A cart is resting on its handle. The angle between the handle and the ground is 14° and the handle connects to the center of the wheel. What are the measures of the arcs of the wheel between the ground and the cart? *Explain*.

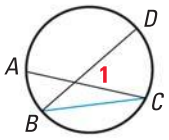
27. PROVING THEOREM 10.11 The proof of Theorem 10.11 can be split into three cases. The diagram at the right shows the case where $\overline{AB}$ contains the center of the circle. Use Theorem 10.1 to write a paragraph proof for this case. What are the other two cases? (*Hint:* See Exercises 31–33 on page 678.) Draw a diagram and write plans for proof for the other cases.



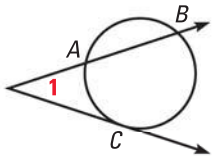
28. PROVING THEOREM 10.12 Write a proof of Theorem 10.12.

GIVEN ▶ Chords $\overline{AC}$ and $\overline{BD}$ intersect.

PROVE ▶ $m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{DC} + m\overset{\frown}{AB}\right)$



29. PROVING THEOREM 10.13 Use the diagram at the right to prove Theorem 10.13 for the case of a tangent and a secant. Draw $\overline{BC}$. *Explain* how to use the Exterior Angle Theorem in the proof of this case. Then copy the diagrams for the other two cases from page 681, draw appropriate auxiliary segments, and write plans for proof for these cases.



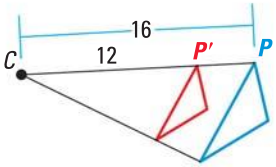
30. PROOF Q and R are points on a circle. P is a point outside the circle. $\overline{PQ}$ and $\overline{PR}$ are tangents to the circle. Prove that $\overline{QR}$ is not a diameter.

31. CHALLENGE A block and tackle system composed of two pulleys and a rope is shown at the right. The distance between the centers of the pulleys is 113 centimeters and the pulleys each have a radius of 15 centimeters. What percent of the circumference of the bottom pulley is not touching the rope?

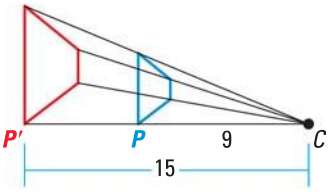
MIXED REVIEW

Classify the dilation and find its scale factor. *(p. 626)*

32.



33.



PREVIEW
Prepare for Lesson 10.6 in Exs. 34–39.

Use the quadratic formula to solve the equation. Round decimal answers to the nearest hundredth. *(pp. 641, 883)*

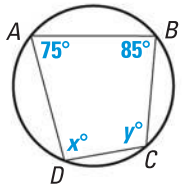
34. $x^2 + 7x + 6 = 0$ **35.** $x^2 - x - 12 = 0$ **36.** $x^2 + 16 = 8x$

37. $x^2 + 6x = 10$ **38.** $5x + 9 = 2x^2$ **39.** $4x^2 + 3x - 11 = 0$

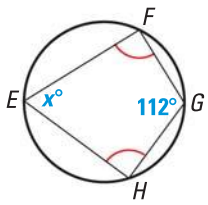
QUIZ *for Lessons 10.4–10.5*

Find the value(s) of the variable(s).

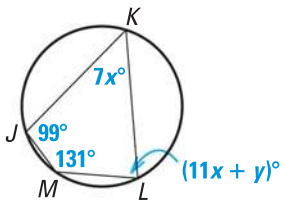
1. $m\overset{\frown}{ABC} = z°$ *(p. 672)*



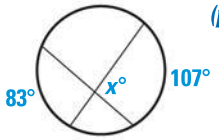
2. $m\overset{\frown}{GHE} = z°$ *(p. 672)*



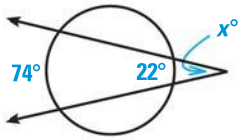
3. $m\overset{\frown}{JKL} = z°$ *(p. 672)*



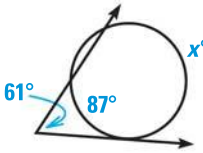
4. *(p. 680)*



5. *(p. 680)*



6. *(p. 680)*



7. MOUNTAIN You are on top of a mountain about 1.37 miles above sea level. Find the measure of the arc that represents the part of Earth that you can see. Earth's radius is approximately 4000 miles. *(p. 680)*